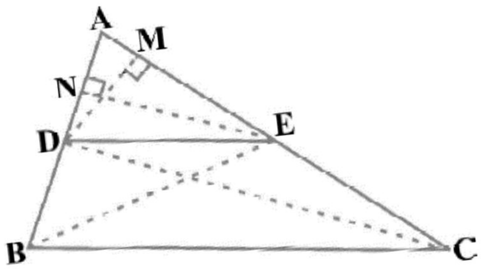
# 10 Standard: Mathematics: Theorems & Formulae

## THEOREM 1
## Basic Proportionality Theorem (BPT) or Thales Theorem

**Statement:** If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.




**Given:** ABC is a triangle. DE || BC and DE intersects AB at D and AC at E.

**To prove**:

$$\frac{AD}{DB}=\frac{AE}{EC}$$

**Construction:** Join B to E and C to D. Draw DN⊥AB and EM⊥AC.

**Proof:**

$$\frac{ar(ADE)}{ar(BDE)}=\frac{\frac{1}{2}ADxEN}{\frac{1}{2}DBxEN}=\frac{AD}{DB}........(1)$$

Similarly,

$$\frac{ar(ADE)}{ar(DEC)}=\frac{\frac{1}{2}ADxDM}{\frac{1}{2}ECxDM}=\frac{AE}{EC}........(2)$$

ΔBDE and ΔDEC are on the same baseDE and between same parallels, DE and BC.

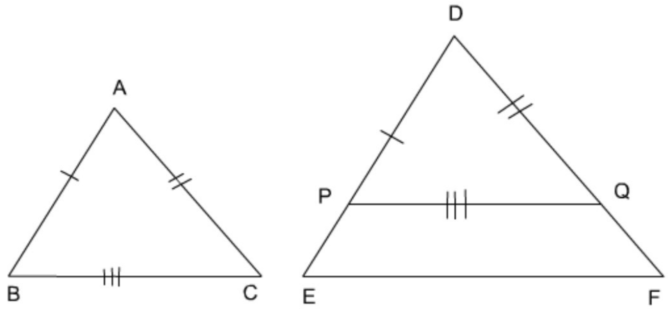
$ar(BDE) = ar(DEC)$ ....... (3)

From (1), (2) and (3)

$$\frac{AD}{DB}=\frac{AE}{EC}$$

## THEOREM 2 (AAA Criterion)

**Statement:** If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.




**Given:**ΔABC and ΔDEF are drawn so that
∠A =∠D, ∠B =∠E and ∠C =∠F
To prove:ΔABC ~ ΔDEF

**Construction:** Draw a line PQ in ΔDEF so that DP = AB and PQ = AC

**Proof:**
In ΔABC and ΔDPQ
AB=DP (By construction)
AC=DQ (By construction)
∠A=∠D (Given)
$\Delta ABC \cong \Delta DPQ$ (SAS Congruence)
∠B = ∠P
∠ E = ∠P (given ∠B=∠E)
PQ || EF

$$\frac{DP}{DE}=\frac{DQ}{DF}$$

$$\frac{AB}{DE}=\frac{AC}{DF}$$ (DP = AB and DQ=AC)

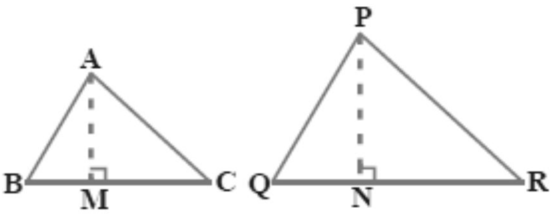
$$\frac{AB}{DE}=\frac{CA}{FD}$$

$$\frac{AB}{DE}=\frac{AC}{DF}=\frac{BC}{EF}$$ and∠A=∠D, ∠B=∠E, ∠C=∠F

ΔABC ~ ΔDEF

## THEOREM 3
## (Areas of similar triangles Theorem)

**Statement:**The ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.




**Given:** ABC ~ PQR

**To prove:**

$$\frac{ar(ABC)}{ar(PQR)}=\left(\frac{AB}{PQ}\right)^2=\left(\frac{BC}{QR}\right)^2=\left(\frac{CA}{RP}\right)^2$$

**Construction:** Draw AM ⊥BC and PN⊥QR

**Proof:**

In $\frac{ar(ABC)}{ar(PQR)}=\frac{\frac{1}{2}x BC x AM}{\frac{1}{2}x QR x PN}=\frac{BC x AM}{QR x PN}$ ....... (1)

In ΔABM and ΔPQN

∠B = ∠Q (Given)

∠M = ∠N (Each is 90°)

Δ ABM ~ ΔPQN (AA Similarity criterion)

$\frac{AM}{PN}=\frac{AB}{PQ}$.......(2)

Δ ABC ~ ΔPQR (Given)

$\frac{AB}{PQ}=\frac{BC}{QR}=\frac{CA}{RP}$........(3)

$\frac{ar(ABC)}{ar(PQR)}=\frac{AB}{PQ}x\frac{AM}{PN}$ (From (1) and (3))

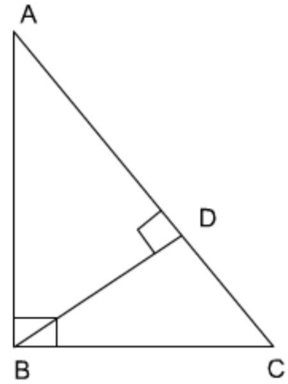
$=\frac{AB}{PQ}x\frac{AB}{PQ}$ (From (2) )

$=\left(\frac{AB}{PQ}\right)^2$

Therefore $\frac{ar(ABC)}{ar(PQR)}=\left(\frac{AB}{PQ}\right)^2=\left(\frac{BC}{QR}\right)^2=\left(\frac{CA}{RP}\right)^2$

## THEOREM 4 (Pythagoras' Theorem)

**Statement:**If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.




**Given:** In ΔABC ∠ABC=90°

**To prove:** $AC^2 = AB^2 + BC^2$

**Construction:** Draw BD ⊥AC

**Proof:**

In ΔABC and ΔADB;

∠A = ∠A (common angle)

∠ADB = ∠ABC (Each equal to 90°)

ΔADB~ΔABC (By AA similarity criterion)

$$\frac{AB}{AC}=\frac{AD}{AB}$$

$AC \times AD = AB^2$ ......... (1)

In ABC and BDC,

∠C = ∠C (common angle)

∠BDC = ∠ABC (Each equal to 90°)

ΔBDC~ΔABC (By AA similarity criterion)

$$\frac{BC}{AC}=\frac{CD}{BC}$$

$AC \times CD = BC^2$ .............(2)

Adding equations (1) and (2), we get;

$$AC \times AD + AC \times CD = AB^2 + BC^2$$

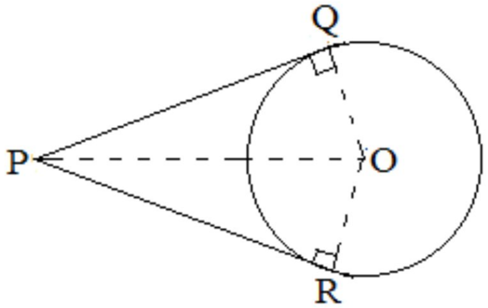
$$Or, AC(AD + CD) = AB^2 + BC^2$$

$$Or, AC \times AC = AB^2 + BC^2$$

$$Or, AC^2 = AB^2 + BC^2$$

## THEOREM 5

**Statement:** The lengths of tangents drawn from an external point to a circle are equal.




**Given:** Two tangents PQ and PR are drawn from a point P to a circle with centre O.

**To prove:** PQ = PR

**Construction :** Join OP, OQ and OR

**Proof:**

OQ⊥PQ (Tangent & radius are perpendicular)

Similarly OQ⊥PQ (Tangent & radius are perpendicular)

In right angled ΔOQP and ΔORP

OQ=QR (Radii of the same circle)

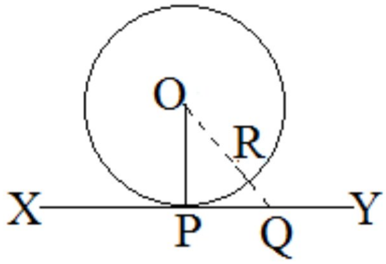
OP = OP(Common side)

ΔOQP ≅ ΔORP (RHS Postulate)

Hence PQ = PR (CPCT)

## THEOREM 6

**Statement:** The tangent at any point of a circle is perpendicular to the radius through the point of contact.




**Given:** A circle with centre O and a tangent XY at a point P of the circle.

**To prove:** OP⊥XY

**Construction:** Take a point Q, other than

**Proof:**

Q is point on the tangent XY, other than the point of contact P.

Q lies outside the circle.

Let OQ intersect the circle at R

OR <OQ (A part is less than the whole).......(1)

OP = OR (radii of the same circle)……………..(2)

OP <OQ (from 1 and 2)

Op is shorter than any other line segment joining O to any point of XY, other than P.

OP is the shortest distance between the point O and the line XY.

The shortest distance between a point and a line is the perpendicular distance

Therefore OP⊥XY

### List of formulae

1) General form of AP
   a, a+d, a+2d, a+3d ...
2) nth term of AP an = a + (n – 1)d
3) Sum of n terms of an AP
   $$Sn = \frac{n}{2}[2a + (n-1)d]$$
   $$Sn = \frac{n}{2}[a + l]$$
4) Sum of first n natural numbers
   $$Sn = \frac{n(n+1)}{2}$$
5) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ it has unique solution (intersecting)
6) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, it has infinitely many solution (Coincident)

8) Cross multiplication method:

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

9) Circumference of circle = $2\pi r$

10) Area of sector of angle = $\frac{\theta}{360} x\ \pi r^2$

11) Length of an arc of a sector = $\frac{\theta}{360}$ $x\ 2\pi r$

12) Distance formula:

13) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

14) Distance from the origin:

$$d = \sqrt{x^2 + y^2}$$

15) Co-ordinates of point P(x,y) divided in the ratio m:n $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

16) Mid-point formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

17) Area of triangle

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

18) Euclid's division lemma: a = bq + r

19) HCF(a,b) x LCM(a,b) = a X b

20) Sum of zeroes = $\alpha + \beta = \frac{-b}{a}$

21) Sum of zeroes = $\alpha + \beta + \gamma = \frac{-b}{a}$

22) Sum of zeroes = $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

23) Product of zeroes = $\alpha\beta\gamma = \frac{-d}{a}$

24) Formula to form a quadratic polynomial $x^2 - (\alpha + \beta)x + \alpha\beta$

25) Division algorithm
P(x) = g(x) x q(x) + r(x)

26) Standard form of quadratic equation
$ax^2 + bx + c = 0$

27) Quadratic formula: x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

28) Discriminant of quadratic equation
$b^2 - 4ac$

29) Nature of roots
If $b^2 - 4ac > 0$, Roots are real & distinct
If $b^2 - 4ac = 0$, Roots are real & equal
If $b^2 - 4ac < 0$, no real roots

30)
sin ɵ = $\frac{Opposite}{Hypotenuse}$; cosec ɵ = $\frac{Hypotenuse}{Opposite}$
cos ɵ = $\frac{Adjacent}{Hypotenuse}$; sec ɵ = $\frac{Hypotenuse}{Adjacent}$
tan ɵ = $\frac{Opposite}{Adjacent}$; cot ɵ = $\frac{Adjacent}{Opposite}$

31) sin ɵ = $\frac{1}{cosec\,\theta}$; cosec ɵ = $\frac{1}{\sin\theta}$
cos ɵ = $\frac{1}{sec\,\theta}$; sec ɵ = $\frac{1}{\cos\theta}$
tan ɵ = $\frac{1}{cot\,\theta}$; cot ɵ = $\frac{1}{\tan\theta}$

32) sin(90 - ɵ) = cos ɵ; cos(90 - ɵ)=sin ɵ
tan(90 - ɵ)=cot ɵ ; cot (90 - ɵ) = tan ɵ
sec(90 - ɵ)= cosec ɵ ; cosec(90-ɵ)=sec ɵ

33) $\sin^2$ ɵ + $\cos^2$ ɵ = 1
$\tan^2$ ɵ + 1 = $\sec^2$ ɵ
1 + $\cot^2$ ɵ = $\mathrm{cosec}^2$ ɵ

34) Mean = $\bar{x} = \frac{\Sigma x_i}{N}$ or $\bar{x} = \frac{\Sigma f_i x_i}{N}$ or
$\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$ or $\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i}$ x h

35) Mode = $l + \left(\frac{f_1 - f_o}{2f_1 - f_o - f_2}\right) \times h$

36) Median = $l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

37) Probability P(E)= $\frac{No.of\ favourable\ outcomes}{Total\ no.of\ possible\ outcomes}$

38) $P(\overline{E}) = 1 - P(E)$

| Solid | LSA/CSA | TSA | Volume |
|---|---|---|---|
| Cylinder | 2πrh | 2πr(r+h) | $\pi r^2 h$ |
| Cone | πr $l$ | πr(r+l) | $\frac{1}{3}\pi r^2 h$ |
| Sphere | $4\pi r^2$ | | $\frac{4}{3}\pi r^3$ |
| Hemisphere | $2\pi r^2$ | $3\pi r^2$ | $\frac{2}{3}\pi r^3$ |

Frustum of cone:

$l = \sqrt{h^2 + (R - r)^2}$

CSA = π (R+r) $l$

TSA = $\pi[(R + r)l + R^2 + r^2]$

Volume= $\frac{1}{3}\pi h(R^2 + r^2 + R \times r)$